Mark Scheme (Results)

Summer 2019

Pearson Edexcel GCE In Decision 2
Paper 6690/01

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\surd$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $*$ The answer is printed on the paper
- ⌐ The second mark is dependent on gaining the first mark
  ⌙

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer

Question Number	Scheme	Marks
1.		
(a)	NNA: D – E – A – F – C – B – D	M1
	36 + 35 + 40 + 37 + 32 + 46 = 226 (km)	A1 **(2)**
(b)	RMST weight = 144 (km)	B1
	144 + 39 + 36 = 219 (km)	M1 A1 **(3)**
		5 marks
Notes		
a1M1:	Nearest neighbour starting at D – must get to at least D – E – A – F – C – Or correct length stated	
a1A1:	Both route and length correctly stated (route must return to D) If double 226 A0. Do not isw.	
b1B1:	CAO for RMST weight (either 144 or 35 + 40 + 37 + 32) – maybe implied by later working	
b1M1:	Adding 39 and 36 (the two least weighted arcs) to their RMST length – this mark maybe implied by the correct value for the lower bound	
b1A1:	CAO – if 219 seen without working then award all 3 marks in (b)	

Question Number	Scheme	Marks
2.		
(a)	<table><tr><td></td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>A</td><td>$25-\theta$</td><td></td><td></td><td>θ</td></tr><tr><td>B</td><td>$3+\theta$</td><td>(17)</td><td>$9-\theta$</td><td></td></tr><tr><td>C</td><td></td><td></td><td>$14+\theta$</td><td>$18-\theta$</td></tr></table> giving <table><tr><td></td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>A</td><td>16</td><td></td><td></td><td>9</td></tr><tr><td>B</td><td>12</td><td>17</td><td></td><td></td></tr><tr><td>C</td><td></td><td></td><td>23</td><td>9</td></tr></table>	M1 A1 **(2)**
(b)	<table><tr><td>Shadow costs</td><td></td><td>17</td><td>16</td><td>10</td><td>14</td></tr><tr><td></td><td></td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>0</td><td>A</td><td>X</td><td>4</td><td>13</td><td>X</td></tr><tr><td>-1</td><td>B</td><td>X</td><td>X</td><td>10</td><td>9</td></tr><tr><td>1</td><td>C</td><td>1</td><td>-3</td><td>X</td><td>X</td></tr></table> <table><tr><td></td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>A</td><td>$16-\theta$</td><td></td><td></td><td>$9+\theta$</td></tr><tr><td>B</td><td>$12+\theta$</td><td>$17-\theta$</td><td></td><td></td></tr><tr><td>C</td><td></td><td>θ</td><td>(23)</td><td>$9-\theta$</td></tr></table> giving <table><tr><td></td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>A</td><td>7</td><td></td><td></td><td>18</td></tr><tr><td>B</td><td>21</td><td>8</td><td></td><td></td></tr><tr><td>C</td><td></td><td>9</td><td>23</td><td></td></tr></table> **Entering cell C2, Exiting cell C4**	M1 A1 M1 A1 **(4)**
(c)	<table><tr><td>Shadow costs</td><td></td><td>17</td><td>16</td><td>13</td><td>14</td></tr><tr><td></td><td></td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>0</td><td>A</td><td>X</td><td>4</td><td>10</td><td>X</td></tr><tr><td>-1</td><td>B</td><td>X</td><td>X</td><td>7</td><td>9</td></tr><tr><td>-2</td><td>C</td><td>4</td><td>X</td><td>X</td><td>3</td></tr></table>	M1 A1
	Optimal as there are no negative improvement indices	A1 **(3)**
(d)	Cost = (£) 1206	B1 **(1)**
		10 marks

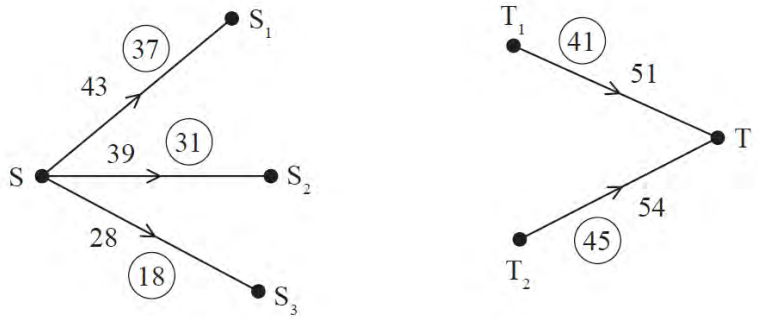
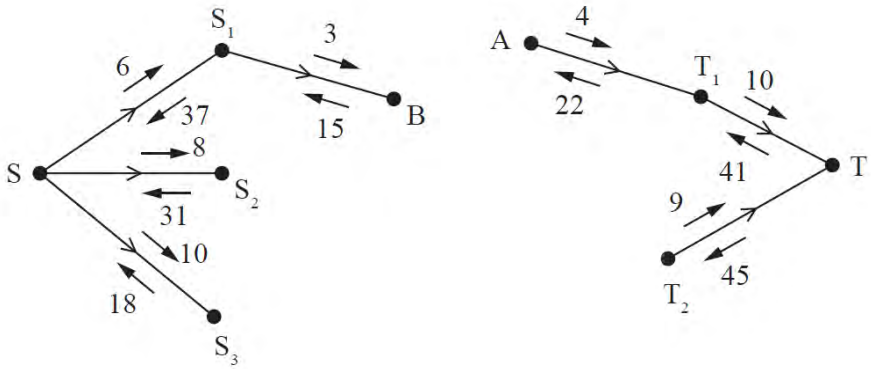
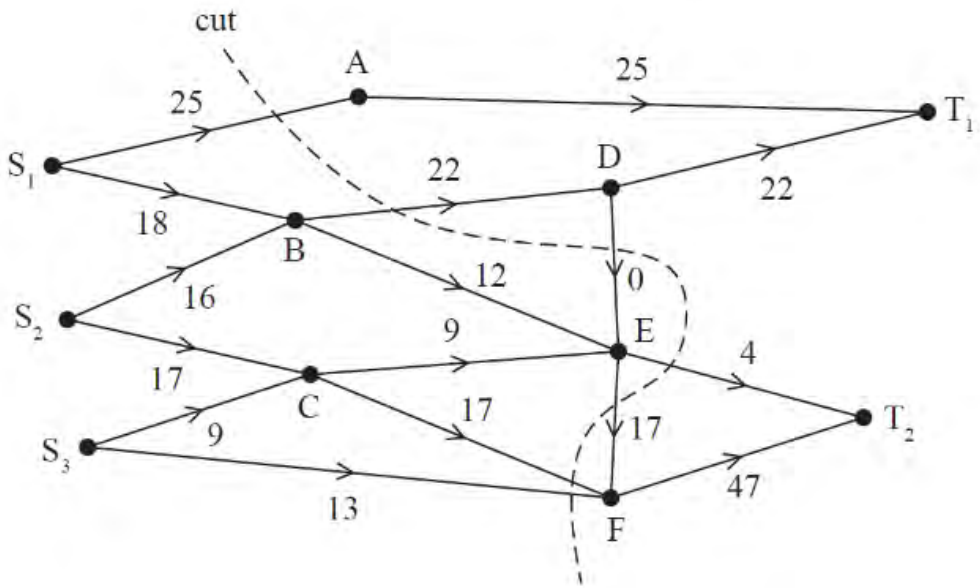
Notes		
a1M1:	A valid route, only one empty square A4 used, thetas balance – some candidates are verifying that A4 is the entering cell (which is fine). For those that start at an incorrect entering cell then the M marks only are available in subsequent parts (unless recovered to the answers given in the scheme)	
a1A1:	Correct route, up to an improved solution (six numbers no zeros) – if there is a zero in cell B3 then A0 unless corrected in part (a)	
b1M1:	Finding 7 shadow costs and at least 6 improvement indices. Condone extra zero IIs.	
b1A1:	Shadow costs and improvement indices CAO (No extra zeros).	
b2M1:	A valid route, their most negative II chosen, only one empty square used, thetas balance	
b2A1:	CSO (for part (b)) – so all previous marks in this part must have been awarded – **including exiting and entering cells stated correctly**(entering is C2 and exiting is C4) – six numbers no zeros	
c1M1:	Finding 7 shadow costs **and** all 6 IIs. Condone extra zero IIs.	
c1A1:	Shadow costs and 6 IIs CAO	
c2A1:	CSO (for part (c)) + optimal + reason	
d1B1:	CAO	

Question Number	Scheme	Marks
3.	Since maximising, subtract all elements from some value ≥ 10	
	e.g. $\begin{bmatrix} 5 & 10 & 3 & 7 & 6 \\ 5 & 7 & 2 & 0 & 9 \\ 6 & 7 & 3 & 1 & 10 \\ 4 & 7 & 4 & 5 & 6 \\ 10 & 8 & 3 & 7 & 8 \end{bmatrix}$	M1
	Reduce rows $\begin{bmatrix} 2 & 7 & 0 & 4 & 3 \\ 5 & 7 & 2 & 0 & 9 \\ 5 & 6 & 2 & 0 & 9 \\ 0 & 3 & 0 & 1 & 2 \\ 7 & 5 & 0 & 4 & 5 \end{bmatrix}$ and then columns $\begin{bmatrix} 2 & 4 & 0 & 4 & 1 \\ 5 & 4 & 2 & 0 & 7 \\ 5 & 3 & 2 & 0 & 7 \\ 0 & 0 & 0 & 1 & 0 \\ 7 & 2 & 0 & 4 & 3 \end{bmatrix}$	M1 A1
	$\begin{bmatrix} 1 & 3 & 0 & 4 & 0 \\ 4 & 3 & 2 & 0 & 6 \\ 4 & 2 & 2 & 0 & 6 \\ 0 & 0 & 1 & 2 & 0 \\ 6 & 1 & 0 & 4 & 2 \end{bmatrix}$ followed by (eg) $\begin{bmatrix} 1 & 3 & 0 & 6 & 0 \\ 2 & 1 & 0 & 0 & 4 \\ 2 & 0 & 0 & 0 & 4 \\ 0 & 0 & 1 & 4 & 0 \\ 6 & 1 & 0 & 6 & 2 \end{bmatrix}$	M1 A1ft M1 A1
	Optimal allocation is F = E, G = D, H = B, I = A, J = C	A1
		8 marks
	Notes	
1M1:	Subtracting from some value which must be ≥ 10 or all values made negative and then adding a value which must be ≥ 10. Condone no more than two errors	
2M1:	Reducing rows and then columns – candidates may combine the two stages of converting from a maximum to a minimum problem and row reduction which is acceptable. (Condone two errors).	
1A1:	CAO	
3M1:	Double covered +e; one uncovered –e ; and one single covered unchanged. 3 lines to 4 lines needed	
2A1ft:	Follow through on their previous table – no errors	
4M1:	One double covered +e; one uncovered –e; and one single covered unchanged. 4 lines needed to 5 lines needed (so getting to an optimal table)	
3A1:	CSO on final table (so must have scored all previous marks)	
4A1:	CAO – this mark is dependent on all M marks being awarded. Allocation must be written clearly. Just labelling zeros in their table scores A0.	
	Special case: Minimising can score at most M0 M1 A1 (Row and column reduction) M0 A0 (Simplification: no 3 zero cover lines to 4). M1 A0 A0 (4 lines to 5). 3/8 marks	

Question Number	Scheme	Marks
4.		
(a)	Row minima: 0, -2, -4 max is 0	M1
	Column maxima: 4, 5, 3 min is 3	A1
	Play safe for Eugene is 1 and for Stephen is 3	A1
	Row maximin (0) $\neq$ Column minimax (3) so not stable	A1 **(4)**
(b)	If Stephen plays safe then Eugene should change from their play safe of option 1 to their option 3 as they will win more against Stephen's play-safe (3 rather than 0)	B1 **(1)**
(c)	Reverses signs in pay-off matrix followed by add 5 to each element. Condone one error.	B1
	Let p_1, p_2, p_3 be the probability of (S) playing 1, 2 and 3 respectively (where $p_1, p_2, p_3 \geq 0$)	B1
	Let V = value of the game (to S)	B1
	Maximise (P =) V	B1
	Subject to:	
	$V - p_1 - 5p_3 + r = 0$ $V - 7p_1 - 4p_2 - 4p_3 + s = 0$ $V - 8p_1 - 9p_2 - 2p_3 + t = 0$ $p_1 + p_2 + p_3 (+u) = 1$	M1 A1 A1
	$(r, s, t, u \geq 0)$	
		(7)
		12 marks
Notes		
a1M1:	Clear attempt to find the Row maximin and Column minimax (either the Row minimums or Column maximums correct or at least four (of the six) values stated correctly)	
a1A1:	Correct Row maximin and Column minimax (dependent on all row mins and column maxs correct) – these could either be stated or clearly shown	
a2A1:	Correct play safe for E (1) and S (3) – not dependent on the previous A mark	
a3A1:	CAO (dependent on all rowmins and colmaxs correct) states $0 \neq 3$ (or row (maximin) $\neq$ col (minimax) as long as 0 is clearly identified as the row maximin and 3 as the column minimax)	
b1B1:	CAO – must mention option 3 and either gain 3 or equivalent in words	
c1B1:	Making all terms non-negative (any addition ≥ 5 is acceptable)	
c2B1:	Defining probability variables	
c3B1:	Defining V	
c4B1:	'maximise' + function/expression	
c1M1:	At least three (of the four) equations or inequalities in V, p_1, p_2, p_3 (with all p_i terms in the first three constraint equations having correct signs for the coefficients) – condone no slack variables for this mark	
c1A1:	CAO - the three constraints involving V and p_i expressed as equations with slack variables	
c2A1:	Probability sum equation correct (allow presence of a slack variable in this equation)	

Question Number	Scheme	Marks
5.		
(a)	<table><tr><td>b.v.</td><td>x</td><td>y</td><td>z</td><td>r</td><td>s</td><td>t</td><td>value</td></tr><tr><td>r</td><td>0</td><td>2</td><td>−3</td><td>1</td><td>0</td><td>0</td><td>30</td></tr><tr><td>s</td><td>−3</td><td>1</td><td>1</td><td>0</td><td>1</td><td>0</td><td>60</td></tr><tr><td>t</td><td>1</td><td>4</td><td>−1</td><td>0</td><td>0</td><td>1</td><td>80</td></tr><tr><td>P</td><td>−2</td><td>−3</td><td>−1</td><td>0</td><td>0</td><td>0</td><td>0</td></tr></table>	M1 A1 B1 **(3)**
(b)	<table><tr><td>b.v</td><td>x</td><td>y</td><td>z</td><td>r</td><td>s</td><td>t</td><td>value</td><td>row ops</td></tr><tr><td>y</td><td>0</td><td>1</td><td>$-\frac{3}{2}$</td><td>$\frac{1}{2}$</td><td>0</td><td>0</td><td>15</td><td>$R_1 \div 2$</td></tr><tr><td>s</td><td>−3</td><td>0</td><td>$\frac{5}{2}$</td><td>$-\frac{1}{2}$</td><td>1</td><td>0</td><td>45</td><td>$R_2 - R_1$</td></tr><tr><td>t</td><td>1</td><td>0</td><td>5</td><td>−2</td><td>0</td><td>1</td><td>20</td><td>$R_3 - 4R_1$</td></tr><tr><td>P</td><td>−2</td><td>0</td><td>$-\frac{11}{2}$</td><td>$\frac{3}{2}$</td><td>0</td><td>0</td><td>45</td><td>$R_4 + 3R_1$</td></tr></table>	M1 A1 M1 A1ft A1 **(5)**
(c)	$P - 2x - \frac{11}{2}z + \frac{3}{2}r = 45$	B1ft
	$r = 0, s = 45, t = 20$	B1 **(2)**
(d)	All values in the (next) pivot column (the z column) are negative and so no further iterations can occur or no viable pivot.	B1 **(1)**
		11 marks

Notes	
a1M1:	Any one row correct (but ignore b.v. column)
a1A1:	All four rows correct (but ignore b.v. column)
a1B1:	b.v. column correct
b1M1:	Correct pivot located (2 in column y), attempt to divide row
b1A1:	Pivot row correct including change of b.v
b2M1:	All values in one of the non-pivot rows correct or one of the non zero and one columns (x, z, r or value) correct following through their choice of pivot from column y
b2A1ft:	Row operations used correctly at least twice, i.e. two of the non zero and one columns (x, z, r or value) correct following through their choice of pivot from column y
b3A1:	CAO – no follow through – all values and row operations correctly stated – allow if row operations given in terms of old row 1 – ignore b.v. column for this mark
c1B1ft:	Follow their profit equation from (b) dependent on scoring both M marks in (b)
c2B1:	CAO (no follow through) for slack variables (r = 0, s = 45, t = 20)
d1B1:	CAO

Question Number	Scheme	Marks
6.		
(a)	Initial flow = 86	B1 **(1)**
(b)		M1 A1 A1 **(3)**
(c)		M1 A1 **(2)**
(d)	E.g. $SS_1AT_1T - 3$ or SS_1AT_1T 3 $SS_1BDT_1T - 3SS_1BDT_1T$ 3 $SS_2CET_2T - 2SS_2BET_2T$ 1 $SS_3CFT_2T - 4$ SS_2CET_2T 1 SS_3CFT_2T 4	M1A1 A1 A1 **(4)**
(e)		M1 A1 **(2)**
(f)	The cut through S_1A, BD, DE, ET_2, EF, CF and S_3F has value of 98	DB1
	Value of the flow is 98 so by max flow – min cut theorem flow is maximal	DB1 **(2)**
		14 marks

Notes		
a1B1:	CAO	
b1M1:	Five arcs added SS_1, SS_2, SS_3, T_1T and T_2T and 2 numbers on each arc, of at least the values shown	
b1A1:	Two correct arcs (flow values and capacities) consistent with their values	
b2A1:	CAO for flow values and capacities (including arrows) consistent with their values	
c1M1:	Two numbers on each arc and at least three arcs or six numbers correct	
c1A1:	CAO do give bod since they might well cross these numbers out	
d1M1:	One valid flow augmenting route found and a value stated	
d1A1:	A second correct flow route	
d2A1:	Three correct flow routes with correct value stated	
d3A1:	CSO flow increased by 12 and no more	
e1M1:	Consistent flow pattern ≥ 93 (check each node). One number only per arc. No unnumbered arcs	
e1A1:	CAO, showing flow of 98, must follow from their routes	
f1B1:	Must have attempted (e) - at least one number on all but one arc, and made an attempt at a cut, either drawn or stated	
f2B1:	CSO - (e) fully correct (showing a correct flow of 98) and a correct cut. Must refer to max flow-min cut theorem – all four words	

Question Number	Scheme	Marks
7.		
(a)	minimax	B1 **(1)**
(b)		M1 A1 (Stage 2) M1 A1 A1 A1 (Stage 2) M1 A1ft A1 (Stage 3) M1 A1 (Stage 4) **(11)**
(c)	Order is A, C, D, B	DB1
	Cost is 25 000 + 50 000 + 30 000 + 45 000 = (£)150 000	M1 A1 **(3)**
		15 marks

Stage	State	Action	Dest.	Value
1	ABC	D	ABCD	65*
	ABD	C	ABCD	55*
	ACD	B	ABCD	45*
	BCD	A	ABCD	55*
2	AB	C	ABC	Max(40, 65) = 65
		D	ABD	Max(35, 55) = 55*
	AC	B	ABC	Max(55, 65) = 65
		D	ACD	Max(30, 45) = 45*
	AD	B	ABD	Max(55, 55) = 55*
		C	ACD	Max(60, 45) = 60
	BC	A	ABC	Max(45, 65) = 65
		D	BCD	Max(50, 55) = 55*
	BD	A	ABD	Max(65, 55) = 65
		C	BCD	Max(35, 55) = 55*
	CD	A	ACD	Max(55, 45) = 55*
		B	BCD	Max(40, 55) = 55*
3	A	B	AB	Max(45, 55) = 55
		C	AC	Max(50, 45) = 50*
		D	AD	Max(55, 55) = 55
	B	A	AB	Max(50, 55) = 55*
		C	BC	Max(45, 55) = 55*
		D	BD	Max(55, 55) = 55*
	C	A	AC	Max(60, 45) = 60
		B	BC	Max(45, 55) = 55*
		D	CD	Max(50, 55) = 55*
	D	A	AD	Max(65, 55) = 65
		B	BD	Max(55, 55) = 55*
		C	CD	Max(60, 55) = 60
4	None	A	A	Max(25, 50) = 50*
		B	B	Max(35, 55) = 55
		C	C	Max(35, 55) = 55
		D	D	Max(30, 55) = 55

Notes		
a1B1:	CAO	
b1M1:	At least three additional rows for the first stage. Value column must contain the values of 55, 45 and 55. Ignore entries in all other columns and condone an error in one value only	
b1A1:	CAO for the first stage (all three additional rows) – **entries in all columns must be correct**	
b2M1:	Second stage – at least 12 rows. Value column must be complete with at least one value correct for each state – ignore entries in all other columns	
b2A1:	Any two states correct in second stage	
b3A1:	Any four states correct in second stage	
b4A1:	Second stage completely correct	
b3M1:	Third stage – at least 12 rows. Value column must be complete with at least one value correct for each state – ignore entries in all other columns	
b5A1ft:	Any two states correct in third stage on the ft from second stage	
b6A1:	Third stage completely correct	
b4M1:	Fourth stage – at least 4 rows. Value column must be complete with at least one value correct – ignore entries in all other columns	
b7A1:	Fourth stage completely correct	
c1B1:	Correct order – dependent on all M marks awarded in (b)	
c1M1:	Correct method for calculating cost for their order (condone not working/answer in thousands)	
c1A1:	CAO	
	Special Cases: The number of rows, for each stage, must be as stated above. Fewer rows for any stage leads to M0. In order to score method marks results in each stage must be have been carried forward from the previous stage. The second accuracy mark is awarded if the correct pair of numbers is seen in each of the 12 rows of stage 2, though incorrect manipulation follows. Minimum Scores at most B0 M1A1 M1A1A0A0 M1A0A0 M1A0 B0M1A0 7/15 marks. Minimin, Maximum, Maximin Scores at most B0 M1A1 M1A1A0A0 M1A0A0 M1A0 B0M0A0 6/15	

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